ENERGY TRANSFER IN THE WTC COLLAPSE

By

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1.0 INTRODUCTION

The tragic events that took place in New York City on September 11th, 2001 have raised a number of questions about the destruction of the buildings comprising the World Trade Center (WTC) but especially the remarkable collapse of the landmark "Twin Towers". Questions range from the emotionally charged – "Who could have done such a thing and why?" - to the more pragmatic: "Why, after appearing to survive two aircraft impacts, did the Twin Towers crumble in a progressive pancake-type collapse of successive floors?" In fact, the dramatic demolition like "take-down" of each tower has prompted some conspiracy-minded observers to suggest that explosives must have been used to initiate each collapse /1/.

A good place to start unraveling the mystery of what caused the Twin Towers to collapse is to investigate the mechanics of the impact and collapse events. This involves following the energy transfer processes from the initial aircraft collisions, through the subsequent fires, to the final collapse and crushing of the steel, concrete and other materials used in the construction of these buildings. In this report we attempt to evaluate the energetics of the impact and collapse events of the September 11th WTC disaster. In this way it is hoped to decide if the observed collapse events could have occurred without the help of explosives or, indeed, without any input from other external sources of energy.

2.0 WTC COLLAPSE TIMES

The collapse times of each of the two WTC towers are very important parameters in the estimation of the energy transfer involved in these events. In this report we define the collapse time, t_c, as the *observed time interval* for more than 95 % of the mass of the WTC tower (WTC 1 or 2) to fall to "ground zero". This, of course, requires a definition of the start of the collapse. Because of uncertainties in the timing of the WTC collapse-initiating ***and*** terminating events, many different values of t_c have been reported; however, the published values (I have seen) all fall in the range 8 – 18 seconds. In addition, Newtonian mechanics dictates a ***minimum*** value for the collapse time, t_c, which is calculated, (allowing for the thickness of each floor), as follows*:

$$\mathbf{t_c = \sqrt{(2h/g)} = \sqrt{\{2(416 - 10)/9.81\}} = 9.1\ seconds}$$

**The calculations included in this report are based on well-documented values for the WTC height, weight and other building specifications as listed in Appendix 1.*

Seismic data recorded at various sites in New York State on September 11th, 2001, show two significant events, the first at about 9:59 a.m. (EDT) and the second, slightly more powerful, at about 10:28 a.m. (EDT). Traces of the seismic events may be viewed at www.ldeo.columbia.edu, and show a lot of "ringing" decay extending over 10 or more seconds. Thus, cursory inspection of the traces suggests that the WTC collapse events were indeed "about 10 seconds" in duration. However, since the correct interpretation of the seismic data is critically important to the analysis of the collapse of the two WTC towers, we will now consider the seismic data in more detail.

The first event, occurring at about 9:59, was the collapse of WTC 2, also called the south tower, (N.B. This was not the first tower to be hit by an aircraft). The north tower, WTC 1, collapsed at about 10:28 and was responsible for the second seismic disturbance. The traces recorded at the Palisades station provide the best seismic data for the events of September 11th, 2001. Because the published traces begin at 9:59:07 (WTC 2) and 10:28:30 (WTC 1), these times are frequently quoted as the actual collapse times. This is erroneous for two reasons. First, it should be noted that the *start* of the major oscillations in the seismic signature of each collapse event corresponds to the ground impact of the main upper section of the towers. As TV coverage of the event shows, this impact occurred about 10 seconds *after* the start of the collapse of each tower. Second, the Palisades seismic data are delayed by about 17 seconds compared to the actual events in New York City because of the travel time for the 34 km distance between the towers and the Palisades seismic station.

The CNN TV coverage of the collapse of the North Tower (WTC 1) provides a very useful time calibration of this event that may be compared with the Palisades seismic data. The CNN TV images show that WTC 1 starts to collapse at 10:28:23. The ground impact of the upper section follows about 10 seconds later at 10:28:33. This is consistent with the Palisades data if we allow 17 seconds for travel time of the seismic waves. Thus, if we treat the Palisades data as if it were recorded at the WTC site, the published seismic trace would now effectively begin at 10:28:13 and the ground impact responsible for the large oscillations of the trace would occur at 10:28:32. These values are in good agreement with the visual result derived from the CNN TV images.

Having made these adjustments to the timelines of the 911 seismic data we are able to conclude that the small ripples in the traces of the WTC collapse events - ripples that ***precede*** the period of large oscillations - represent the *first stage of collapse* as defined more precisely below. The seismic signal for this first stage is small, as would be expected, since kinetic energy is being transmitted to the ground only through the steel support structure. Furthermore, a significant fraction of this kinetic energy is being absorbed as the energy needed to buckle and crush the structural elements of the buildings. The major seismic signal of each collapse is generated by the ground impact of falling debris, and constitutes what we will call *a second stage of collapse*. Given the above considerations and a careful evaluation of the seismic data, it is estimated that *the first stage of collapse* took 11.3 ± 1.5 seconds for each WTC tower. We will show in the following Section that *the second stage of collapse* added 1 – 2 seconds to the total collapse times.

3.0 MOMENTUM TRANSFER THEORY OF THE WTC COLLAPSE

Direct observation tells us that the Twin Towers both collapsed in a time a few seconds greater than the 9.1 second free fall time of an object dropped from a height of 416 meters onto a base about 10 meters high. We now present a simple momentum transfer theory that may be used to calculate values of t_c for each of the WTC towers.

We begin by noting that live television coverage of the events of 911 show that WTC 1 and WTC 2 collapsed from the structural failure of severely damaged *upper* floors located close to the aircraft impact points. These impacts are centered at floor 96 for WTC 1 and floor 81 for WTC 2. Thus we assume that a mass of 14 (more or less intact) floors fell onto the remaining 96 (more or less intact) lower floors of WTC 1 and 29 upper floors fell onto 81 lower floors in WTC 2. For the general case of n floors collapsing we define a collapsing mass M_c :

$$\mathbf{M_c = n\, m_f} \quad \text{…………………….} \mathbf{(1)}$$

where $\mathbf{m_f}$ is the mass of *one* WTC floor, assumed to be 1/110 the mass of an entire WTC tower, namely $m_f = (510{,}000{,}000 / 110)\ \text{kg} \approx 4{,}636{,}000\ \text{kg}$

We consider the initiating event of a WTC tower collapse to be the failure of crucial steel support structures at the appropriate upper floor level of the building, followed by the free fall of the entire upper block of n floors through a distance h_f = one floor height = 3.7 meters. It is readily determined using the relation $v = \sqrt{\{2gh)}$ that the descending upper block impacts the floor below at a velocity of 8.5 m/s. The law of conservation of momentum states that:

$$m_1 \times v_1 = m_2 \times v_2$$

We will use this law for the *non-elastic* collision where the colliding masses essentially merge into a single mass that continues to descend. For the simplest case of one floor collapsing onto an identical floor,

$$m_1 = m_f \, ; \, m_2 = 2m_f$$

Hence,

$$m_f \times v_1 = 2m_f \times v_2$$

or,

$$v_2 = \tfrac{1}{2} v_1$$

Thus, for this simple case, the merged floors continue their downward path with a velocity *equal to half the initial impact velocity*. In using the law of conservation of momentum in this way it is tacitly assumed that the impulse delivered by the impact is sufficient to rupture not only the vertical columns supporting the impacted floor but also the steel truss supports that span the gap between the outer perimeter wall and the inner core of the building.

We now apply this simple model to the WTC collapse. We assume that both WTC building collapses began with an upper block of **n** floors collapsing onto a series of lower floors as in the "domino effect". We shall refer to this process as the *first stage of collapse*. For this stage, (see equation 1), we have an initial mass $\mathbf{nm_f}$ falling onto the floor below and becoming mass $\mathbf{(n+1)m_f}$. This new, enlarged, block of floors descends with velocity $v_2 = \{n/(n+1)\}v_1$ through a distance h_f at which point it strikes the floor below and becomes mass $\mathbf{(n+2)m_f}$ moving at velocity $\{n/(n+2)\}v_2$, and so on. This implies a first stage collapse sequence for WTC 1: all floors from 110 to 96 (= 14 floors) collapse onto floor 95; all these floors collapse onto 94 → 93 → 92 and so on to 3→ 2 → 1; for WTC 2 all floors from 110 to 81 (= 29 floors) follow the same sequential process.

At the end of each of these collapse events we envision a *second stage of collapse* involving the destruction of the upper block of the WTC buildings: for WTC 1 the 97th floor, plus all floors above, collapse onto the pile of rubble topped by floor 96; this is followed by floor 98 (plus all floors above) collapsing onto floor 97 and so on. The 2nd stage sequence for WTC 1 ends with floor 110 collapsing on to all lower floors. For WTC 2 the 2nd stage involves floor 82 collapsing onto floor 81, followed by 83, 84, etc, collapsing on to the pile of rubble until floor 110 collapses onto all lower floors.

An Excel program using our momentum transfer theory has been written. For the first stage of each of the two WTC collapse events we find the following values of t_c:

WTC 1	WTC 2
t_c (1st stage of collapse) = 11.6 sec	t_c (1st stage of collapse) = 9.7 sec
Final impact velocity, v = 51.2 m/s	Final impact velocity v = 50.7 m/s

The 1st stage collapse times given above are in reasonable agreement with the observed collapse times discussed in Section 2.0 and account for most of the magnitude of t_c. Nevertheless, it is straightforward to add a small correction to t_c to include a 2nd stage of collapse. For the stage-2 collapse it is reasonable to assume that momentum transfer does *not* effect the collapse time. Thus, for WTC 1, we must simply calculate the time for the top 14 floors to descend to ground zero; similarly, for WTC 2, we need to calculate the time for the 29 upper floors to pile up.

The 2nd stage collapse calculation uses the well-known equations for free fall of an object in the earth's gravity:

$$v_2 = \sqrt{(v_1^2 + 2gnh_f)} \quad \text{and} \quad t = 2nh_f / (v_1 + v_2)$$

where n is the number of floors, (n =14 for WTC 1 and n = 29 for WTC 2).

From the end of the 1st stage collapse we have the initial velocities for the 2nd stage:

WTC 1: $v_1 = 51.2$ m/s and WTC 2: $v_1 = 50.7$ m/s

From which we find:

WTC 1	**WTC 2**
t_c (2nd stage of collapse) = 1.0 sec	t_c (2nd stage of collapse) = 1.8 sec
Final impact velocity, v = 60.3 m/s	Final impact velocity v = 68.4 m/s

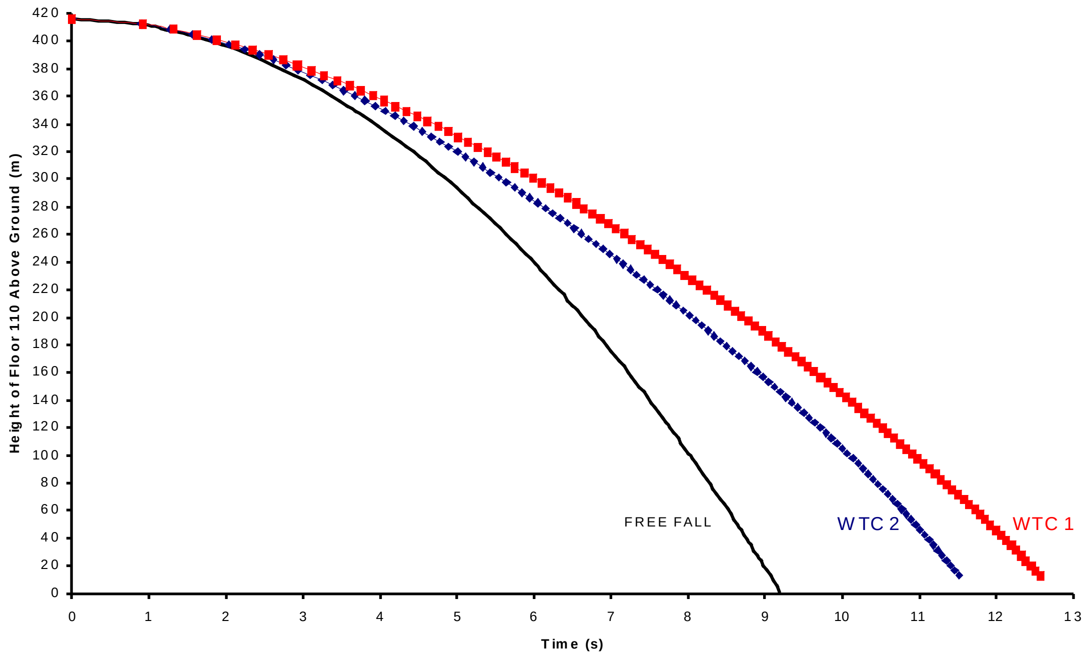
We are now in a position to determine the **total collapse times** by adding the 1st and 2nd stage times given above. The **calculated values** are:

WTC 1	**WTC 2**
t_c = (11.6 + 1.0) sec = 12.6 sec	**t_c = (9.7 + 1.8) sec = 11.5 sec**

As noted above, these WTC 1 and 2 collapse times are based on a stepwise momentum transfer calculation for each floor. Thus, it is possible to follow the detailed floor-by-floor progress of the collapse of each WTC tower as shown in Figure 1. This figure includes a plot of the free fall of an object from 416 meters, which takes about 9 seconds, and clearly illustrates how the delay in the fall of each tower develops.

We note in concluding this Section that the values for t_c given above represent the calculated values for the time of collapse of the WTC towers *neglecting the energy required to crush or otherwise destroy the support structure of each floor*. This energy, which we will call E_1, is considered in detail in Section 4.2. For now it is sufficient to note that the collapse times calculated *without* allowing for E_1 are already in reasonable agreement with the observed collapse times. This suggests that E_1 is relatively small compared to the kinetic energy associated with the falling blocks of floors; let us now place this qualitative prediction on a quantitative basis. To do this we must calculate the energies involved in each stage of the WTC collapse and then correct for the *resistance* offered by each floor.

Figure 1: WTC TOWERS 1 and 2 COLLAPSE PROFILES



4.0 ENERGETICS OF THE WTC COLLAPSE

Now that we have a reliable initial estimate of the collapse times of WTC 1 and 2 we are in a position to look more closely at the mechanics of the collapse events. As a first approximation we have considered each collapse to involve an inelastic collision between *unsupported* (or weakly supported!) blocks of floors. However, to more precisely model the physics of the WTC collapse events, we need to consider the bending and fracturing energy, E_1, that must be expended to collapse each floor. The magnitude of E_1 is discussed in detail in Section 4.2, but first let us consider the kinetic energy available from the collision of floors as predicted by simple inelastic collision theory.

4.1. KINETIC ENERGIES FROM INELASTIC COLLISION THEORY

Consider a *totally inelastic collision* involving a block of N floors, each floor of mass m_f falling onto a single unattached floor. The velocity before collision, u, is related to the velocity after collision, v, by the law of conservation of momentum:

$$Nm_f \times u = (m_f + Nm_f) \times v$$

Hence,

$$v = u \{N/(1 + N)\}$$

The initial kinetic energy, T_i, of the falling block of N floors is given by:

$$T_i = ½\, Nm_f\, u^2$$

The kinetic energy after the first collision, T_a, is given by:

$$T_a = ½\, (m_f + Nm_f)\, v^2$$

Hence,

$$T_a = ½\, m_f\, (1 + N) \times \{u^2\, N^2/(1 + N)^2\}$$

or,

$$T_a = ½\, m_f\, u^2\, N^2/(1 + N)$$

Let the kinetic energy that is "lost" in the inelastic collision be represented by Q; this energy is ultimately dissipated as heat.

$$Q = T_i – T_a$$

Substituting for T_i and T_a we find,

$$Q = ½\, Nm_f\, u^2 – ½\, m_f\, u^2\, N^2/(1 + N)$$

or,

$$Q = ½\, m_f\, u^2\, \{N/(1 + N)\}$$

But the initial kinetic energy T_i is equal to $½\, Nm_f\, u^2$ so the fractional conversion, f_c, of kinetic energy to heat is simply,

$$f_c = Q/T_i = 1/(1 + N)$$

This is an important result because it shows that for a series of inelastic collisions, which we believe is a good first approximation to the collapse of the WTC towers, a significant fraction of the kinetic energy generated during the collapse is lost as heat. To fully illustrate this point consider the following example:

For *one* WTC floor collapsing onto a floor below, the kinetic energy immediately before impact is

$$T_i = ½\{510,000,000/110\} \times (8.5)^2 \text{ Joules}$$

$$T_i = 1.67 \times 10^8 \text{ J}$$

The kinetic energy of the combined floors immediately after impact is

$$T_a = ½\{2 \times 510,000,000/110\} \times (8.5/2)^2 \text{ Joules}$$

or,

$$T_a = ½ T_i = Q$$

Thus, in the case of one floor collapsing onto the floor below, 50 % of the kinetic energy is dissipated as heat! However, we have shown that as we increase the number of collapsing floors, the fractional loss of kinetic energy, f_c, decreases as 1/(1 + N), where N is the number of falling floors. Since the WTC 1 collapse consisted of 14 floors impacting the floor below, and the WTC 2 collapse involved 29 floors impacting the floor below, we have the kinetic energy before impact,

$$T_i \text{ (WTC 1)} = 14 \times 1.67 \times 10^8 \text{ J} = 23.4 \times 10^8 \text{ J}$$

$$T_i \text{ (WTC 2)} = 29 \times 1.67 \times 10^8 \text{ J} = 48.4 \times 10^8 \text{ J}$$

The kinetic energy lost as heat, which we shall call Q (WTC 1) or Q (WTC 2), is $f_c \times T_i$ where $f_c = 1/(1 + N)$. Hence,

$$Q \text{ (WTC 1)} = 1/(1 + N) \times T_i \text{ (WTC 1)} = 14/15 \times 1.67 \times 10^8 \text{ J} = 1.56 \times 10^8 \text{ J}$$

$$Q \text{ (WTC 2)} = 1/(1 + N) \times T_i \text{ (WTC 2)} = 29/30 \times 1.67 \times 10^8 \text{ J} = 1.61 \times 10^8 \text{ J}$$

A comparison of these Q values with the initial kinetic energies, T_i (WTC 1) and T_i (WTC 2), shows that a relatively small fraction of the available energy, (6.7 % for WTC 1 and 3.3 % for WTC 2), is converted to heat by the first impact of the upper blocks of floors. Because the fractional conversion of energy to heat is even smaller for subsequent impacts, most of the kinetic energy of collapse is conserved from one floor impact to the next. Thus *a rapid self-sustaining total collapse of the towers* is an inevitable consequence of first order momentum transfer theory.

One circumstance that would change this catastrophic sequence of events would be through the storage of kinetic energy as elastic strain energy in the floor supports. Certainly, the architectural design of the WTC called for the floors to be well supported by many high strength steel box columns. Furthermore, the strength and elasticity of these structures are not explicitly considered in our first order calculations. We therefore need to examine the role of the floor supports in the WTC collapse and determine how they might influence the collapse times.

4.2. IMPACT ENERGY REQUIRED TO COLLAPSE ONE WTC FLOOR

A crucial question that is frequently asked concerning the collapse of the WTC towers is why did the localized damage near the impact levels in WTC 1 and 2 cause the collapse of the entire buildings? In order to answer this question we need to move beyond our simple momentum transfer collision theory and consider *how much energy is needed to bring about the collapse of one floor. We call this energy E_1.* Once we have a reliable estimate for E_1 we will be in a position to compare it to the kinetic energy, T_i, associated with the free fall of particular blocks of floors. If T_i is found to be significantly larger than E_1, a self-sustaining total building collapse is possible. If the converse is true, only a collapse of floors severely damaged by the initial aircraft impact is possible. A comparison of our estimates of E_1 and T_i is made in Section 6.0. For now we will focus on a reliable determination of E_1.

Unfortunately there appears to be no simple way to calculate E_1 from first principles since the collapse of just one floor of a WTC tower is an extremely complex process involving the bending and fracturing of numerous support structures. In addition, because each WTC tower began collapsing at a floor close to the impact point of a Boeing 767 aircraft, it is necessary to quantify the local damage caused by these impacts. The *external* pre-collapse damage to each WTC tower is clearly visible in the photographs included in Figures 2-16 and 2-27 of the FEMA *WTC Building Performance Study* /2/. These photos show that up to 36 exterior columns were severed in regions surrounding the impact point of each aircraft. However, the FEMA images provide very little information on the extent of damage to the 47 *interior* box columns that constitute the structural core of each WTC tower.

In spite of these uncertainties, some estimates of the magnitude of E_1, (the energy needed to bring about the collapse of one floor), have been made. For example, Z. Bažant et al. at Northwestern University, Illinois, have estimated that the maximum plastic energy dissipated by the collapse of one floor, i.e. our quantity E_1, is approximately equal to 5.0×10^8 J. Unfortunately Bažant et al. do not give a detailed exposition on how this value for E_1 was derived, stating only that it is based on “approximate design calculations” for one WTC tower /3/.

Another calculation that may be used to estimate E_1 was published by G.C. Lee et al. in a MCEER Special Report /4/. Lee et al. assume that 36 exterior columns on WTC 1 were destroyed by the Boeing aircraft impact and conclude (without giving computational details) that the energy absorbing capacity of these damaged columns “does not exceed 7230 kips-ft” or about 10^7 J. Based on this estimate, and remembering that one complete floor has 236 exterior columns, it follows that the exterior columns comprising one floor of a WTC tower have an impact energy absorbing capacity of about 7×10^7 J. From the relative cross-sectional area of a core column (0.1236 m^2) compared to an exterior column (0.0184 m^2), we estimate that the effective strength of the core columns is about 6.7 times higher than the effective strength of the exterior columns. A consideration of the collapse of the 47 core columns therefore adds about 9×10^7 J of energy absorbing capacity. Thus, based on Lee's calculations, the total energy absorbing capacity of the structural supports of one floor of the WTC is estimated to be about 1.6×10^8 J, which we equate to our quantity E_1 while noting that this estimate is significantly *lower* than Bažant's value of 5.0×10^8 J. However, it appears that Lee's results are based on very rough estimates of the energies involved so that the level of agreement with Bažant's estimates is as good as might be expected in view of the approximations involved.

A much better estimate for E_1, and one that is based on a very detailed analysis of the aircraft impact events, may be derived from a paper published by T. Wierzbicki et al. at MIT /5/. These authors have calculated the energy dissipated by the wing of a Boeing 767 cutting through the exterior columns of a WTC tower and report a value equal to1.139×10^6 J *per column*. On this basis, 2.69×10^8 J would be require to cut through all 236 exterior columns supporting one WTC floor. If we now assume, as previously discussed, that the yield strength of the *core columns* is about 6.7 times higher than the yield strength of the *exterior columns*, we estimate that an additional 3.60×10^8 J are required to collapse the 47 *core columns* supporting each floor. Thus, based on T. Wierzbicki et al. calculation, we estimate a total of 6.29×10^8 J of impact energy was required to collapse one WTC floor, a value that is remarkably close to Bažant's estimate of 5.0×10^8 J for the plastic energy dissipated by the collapse of one floor.

The fact that the values of E_1 derived from Wierzbicki's and Bažant's studies are quite similar is very significant because these author's calculations were actually undertaken for two different impact events: (i) The collision of a Boeing aircraft with one floor of a WTC tower, and (ii) The collapse of a block of WTC floors onto the floor below. Thus Wierzbicki considers floor support failure under *lateral impact loading* while Bažant's considers the failure of the floor supports under *axial impact loading*. The fact that the energy calculated in each of these cases is about the same suggests that the energy dissipated in a floor collapse is relatively insensitive to the mode of failure of the support structures. This is a common observation in studies of collisions of large objects involving complex structures such as aircraft, automobiles, trains, and ships.

4.3 BOEING 767 AIRCRAFT IMPACT ENERGY DISSIPATION

Now that we have an estimate of the energy needed to collapse the support structures of one WTC floor we are in a position to evaluate the post-impact dissipation of the kinetic energy supplied to each of the twin towers by the impacting Boeing 767 aircraft. There have been many estimates of the kinetic energy, E_c, involved in each of the plane crashes, however, for simplicity, we will use a single value based on an assumed aircraft mass of 124,000 kg and a velocity at the moment of impact of 220 m/s. With this mass and velocity, the aircraft impact kinetic energy is equal to,

$$E_c = ½ \times 124{,}000 \times (220)^2 \text{ J} = 3.0 \times 10^9 \text{ J}$$

In Section 4.2 we showed that the energy needed to destroy the structural supports of a WTC tower by an aircraft impact was about 0.6×10^9 J. We must therefore look for additional impact energy sinks to account for the dissipation of the remaining 2.4×10^9 J of supplied kinetic energy. Two important energy sinks are the elastic strain energy dissipated by the sway of the recoiling building and the energy dissipated by the destruction of the impacting aircraft.

(i) Building Sway

An estimate of the extent of building sway following an aircraft collision may be made on the basis of simple momentum transfer theory. Let the mass of the aircraft be represented by M_a and its impact velocity by V_a. We consider the impacted building to have an effective mass M_b, equal to the mass of the upper 1/3rd of the structure and assume a uniform mass distribution for the building. With a recoil velocity immediately after impact represented by V_b, conservation of linear momentum dictates that,

$$V_b = \{ M_a / M_b \} \times V_a$$

Using our previous values for these quantities we have,

$$V_b = \{124{,}000 / 170{,}000{,}000\} \times 220 \text{ m/s}$$

$$V_b = 0.16 \text{ m/s}$$

An estimate of the deflection of the top of a WTC tower induced by a dynamic load may be obtained from this recoil velocity by using the elastic response theory presented by W. Schueller /6/. First we note that the natural period of vibration of a WTC tower, T, is equal to 11 seconds. Now if V_b is taken to be the *maximum* velocity of the top of the tower at its central position, it follows that the amplitude of vibration, Y, which is also the maximum displacement, is given by,

$$Y = \{ V_b \times T \} / 2\pi$$

Substituting our values of V_b and T into this equation we find that Y is only 0.28 meters. A WTC tower at a deflection 0.28 meters has converted all the initial kinetic energy of the swaying building to elastic strain energy that is ultimately dissipated as heat. This energy, equal to $\frac{1}{2} M_b V_b^2$ or 2.18×10^6 J, is less than 0.1 % of the initial kinetic energy of the impacting aircraft.

(ii) Destruction of the Impacting Aircraft

A remarkable feature of the aircraft collisions with the twin towers was that each Boeing 767 appeared to enter the façade of each building with relatively little visible impact damage. Additionally, only limited amounts of aircraft debris subsequently emerged from the opposite side of each tower. These observations indicate that the outer perimeter wall of the twin towers offered a relatively “soft target” to the impacting aircraft wings and fuselage while the inner core of the building represented a “hard target” that rapidly brought the aircraft to a complete stop. If we assume that 36 exterior columns were severed by the aircraft strike, and take T. Wierzbicki’s value of 1.139×10^6 J as the energy required to sever one exterior column, we conclude that 4.1×10^7 J of energy was dissipated at the perimeter wall from an aircraft possessing an initial 3.0×10^9 J of kinetic energy. This is equivalent to a velocity reduction of only 1.5 m/s, namely, from 220 m/s to 218.5 m/s.

Let us now consider a Boeing 767 aircraft moving at 218.5 m/s that has penetrated the perimeter wall of a WTC tower and impacts the inner core wall. A Boeing 767 aircraft is 49 meters long and the average distance from the perimeter wall to the core wall is 17 meters. This implies that the rear end of the impacting aircraft must have traveled 32 meters in order to completely penetrate the tower in the manner observed. We will assume that the striking aircraft was brought to a full stop in this distance. The average velocity during this period of deceleration is ½ of 218.5 m/s or 109.25 m/s and the impact time is therefore 0.293 seconds. If we consider the core wall to be completely rigid and that the impacting aircraft is subject to a constant reaction load F_a, the deceleration of the aircraft is simply (218.5/0.293) $m.s^{-2}$ = 746 $m.s^{-2}$ equivalent to 76 g’s. From Newton’s Law we then infer that,

$$F_a = 124{,}000 \times 746 \text{ Newtons} = 92.5 \text{ MN}$$

J. D. Riera, in his classic paper on aircraft impact into rigid structures, Ref /7/, has estimated that the maximum buckling load necessary to crush the fuselage of a large commercial aircraft is less than 10 MN, so we have more than enough force to crush a Boeing 767. In a more recent assessment of Riera’s approach, A. K. Kar (Ref /8/) has estimated that a Boeing 707 weighing 91,000 kg impacting a rigid structure at 104 m/s would be subject to a *peak* load of 92 MN.

To model the WTC aircraft impacts along the lines used by Riera and Kar we will assume that for crushing up to the mid-point of the aircraft the buckling load exerted on the Boeing 767 obeys Hooke’s law so that $F_a = k \times x$, where k is the spring constant and x is the length of aircraft crushed. We also assume that at x = 24.5 meters, F_a is at a maximum value of 100 MN giving k = 4.08 MN/m. For crushing beyond x = 24.5 meters we assume that F_a declines in a linear fashion with k = – 4.08 MN/m. It follows that the total energy dissipated in crushing a Boeing 767 aircraft is $4.08 \times (24.5)^2 \times 10^6$ J = 2.45×10^9 J. This value is consistent with the other energies involved in the aircraft impacts on the WTC towers as discussed above.

5.0 ENERGY TO CRUSH THE WTC CONCRETE

One of the most intriguing aspects of the collapse of the WTC towers was the production of vast swirling clouds of fine particulate material as the buildings fell. Subsequent analysis of the particulate material that settled out and formed a grayish-white dusting over much of Lower Manhattan showed it to be mainly a mixture of crushed gypsum wallboard, glass fiber insulation and concrete, some of which was in the 60 ± 20 μm (micron) size range. As shown in Appendix 1 each WTC tower contained about 48,000,000 kg of concrete flooring. It is suggested in Ref /1/ that the energy released by the collapse of the WTC towers was insufficient to crush such a large amount of concrete into such a fine dust. This question will be investigated in this Section.

In Appendix 2 we present a simple formalism to calculate the energy needed to crush the concrete floors in each tower to 60 μm particles and determine a value of 3.2×10^{11} J. For the more general case of the energy needed to crush concrete to particles measuring d μm across, it is readily shown that the surface area of 1 kg of such particles is 4000/d(μm) m^2, and the energy needed to crush concrete to this particle size is $100 \times \{4000/d(\mu m)\}$ joules/ kg.

The total kinetic energy generated by the collapse of one WTC tower was about 10^{12} J. It was estimated in Section 4.2 that an average of about 10^9 J of energy was expended in collapsing each WTC floor. Thus about 10^{11} J of energy was expended in collapsing all the floors in a WTC tower. This leaves about 9.0×10^{11} J of energy to crush the wallboard, insulation and concrete in each tower. We can therefore conservatively assume that at least 5×10^{11} J of kinetic energy was available to crush the WTC concrete.

Let's now consider the beginning of the 1st sage of the collapse of each tower. For WTC 1 we will take as an example 14 floors, and for WTC 2, 29 floors impacting the floor below with a maximum velocity of 8.6 m/s. It follows that the kinetic energy on impact was $½ \times 14 \times (510{,}000{,}000/110) \times (8.6)^2$ joules = 2.4×10^9 J for WTC 1, and the K.E. was $½ \times 29 \times (510{,}000{,}000/110) \times (8.6)^2$ joules = 5.0×10^9 J for WTC 2. If we assume 50 % of this energy was available to crush concrete, we have 1.2×10^9 J available for WTC 1, and 2.5×10^9 J available for WTC 2. This is sufficient to crush the concrete on the impacted floor to 175 μm particles.

Consider now the newly formed mass of (14 + 1) floors of WTC 1, and (29 + 1) floors of WTC 2, impacting on the floor below. Because of momentum transfer, the impact velocities are slightly *lower* than the 8.6 m/s impact speed for the first floors hit: 8.1 m/s for WTC 1, and 8.3 m/s for WTC 2. The maximum kinetic energy prior to impact is $½ \times 15 \times (510{,}000{,}000/110) \times (8.1)^2$ joules = 2.3×10^9 J for WTC 1, and $½ \times 30 \times (510{,}000{,}000/110) \times (8.3)^2$ joules = 4.8×10^9 J for WTC 2. This is essentially the same result as the previous impact calculation and the kinetic energy released is therefore also sufficient to crush the concrete on the impacted floor to 175 μm particles.

However, if we continue this method of calculation to the 3rd, 4th, 5th, etc, impacts on successively lower floors the kinetic energy increases substantially. For example, for the 5th impact of the less energetic WTC 1 collapse, the kinetic energy is about 1×10^{10} J which is sufficient to crush the concrete on the impacted floor to sub-20 μm particles.

Finally, we will calculate the energy needed to crush ***all*** the concrete in a single WTC tower (= 48,000,000 kg) to particles of a specified size. As we have noted before, the energy required to crush all of the concrete in one tower to 60 μm particles = 3.2×10^{11} J which is only slightly less than the 5×10^{11} J of kinetic energy available. However, the energy required to crush concrete to 100 μm particles is 1.9×10^{11} J, which is well within the crushing capacity of the available energy. Hence it is theoretically possible for the WTC collapse events to have crushed more than 90 % of the floor concrete to particles well within the observed particle size range.

6.0 DISCUSSION

The Question of the Collapse Times

In order to understand how the world famous "Twin Towers" fell in the aftermath of two commercial aircraft impacts we need to understand the underlying physics of the processes involved. We have therefore developed a simple collision theory based on a detailed analysis of the well known, and much discussed, collapse events.

As a first approximation, a momentum transfer theory of the collapse of each building was developed (in Section 3) to account for the observed collapse times of about 12 seconds. The first order calculation considers the impact of descending blocks of floors to be totally inelastic and ignores the energy needed to buckle and fracture the support structures of each tower. The resulting calculated values of the collapse times, t_c, (12.5 seconds for WTC 1 and 11.5 seconds for WTC 2), are already quite close to the observed collapse times but such agreement could, of course, be fortuitous. In Section 4, we next consider the influence of energy dissipation within the collapsing structures. The sources of impact energy loss include the energies needed to crush the impacting aircraft and to destroy support structures such as the core box columns within each tower. To this end we have used data taken from published sources to calculate the magnitude of the energies involved in the collapse events.

Table 1 presents a summary of the essential results of these calculations. All energy values are quoted as *averages of each aircraft impact and each tower collapse* and are expressed in Gigajoules = 10^9 J.

Table 1: A Summary of the Average Energies Involved in the Initial Collapse Events

	Aircraft Impact (Gigajoules)	Upper Block Collapse (Gigajoules)
Input Energy (Kinetic or Gravitational)	3.0	3.6
Energy to Crush Boeing Aircraft	2.5	-
E_1 = Energy to Collapse Floor Supports	0.6	0.6
Energy per Floor to Crush Concrete to 60 μm	-	2.9

It is very encouraging that the energies reported in Table 1 show an acceptable balance between the energy inputs to the towers and the energies dissipated by the destruction of the aircraft and the WTC floor supports. For our calculations to show this level of internal consistency argues well for their validity. And while we acknowledge that the energies in question could be determined with greater precision by more detailed calculations, we would suggest that the values in Table 1 are a useful first approximation.

With an estimate of the magnitude of the major energy sinks listed in Table 1, we are in a position to assess their effect on the WTC collapse time, t_c, by including the effects of collapse energy loss into our first order calculation of t_c. This has been done by subtracting an assumed value for the collapse energy, E_1, from the input kinetic energy, T_i, and re-calculating the post-impact velocity using the fact that,

$$v = \sqrt{\{ 2E_r / M)}$$

where M is the mass of the falling block of floors and,

$$E_r = T_i - E_1$$

N.B. it was shown in Section 4.1 that for the number of floors involved in each tower collapse,

$$T_i \text{ (WTC 1)} = 2.34 \times 10^9 \text{ J}; \qquad T_i \text{ (WTC 2)} = 4.84 \times 10^9 \text{ J}$$

We have re-calculated the descent velocity after the impacts on every floor and determined a revised collapse time that now includes the effects of the energy lost in crushing the support structures. Rather than restrict our calculation to one value of E_1, say 0.6×10^9 J as given in Table 1, we have carried out the calculation with E_1 treated as a variable parameter in the range zero to 2.4×10^9 J. Some of the key results of these calculations are shown in Figure 2. Based on an assumed value of 0.6×10^9 J for E_1 we have the following revised estimates for t_c:

WTC 1	**WTC 2**
Previously ($E_1 = 0$) $t_c = 12.6$ sec	**Previously ($E_1 = 0$) $t_c = 11.5$ sec**
Revised ($E_1 = 0.6 \times 10^9$ J) $t_c = 12.8$ sec	**Revised ($E_1 = 0.6 \times 10^9$ J) $t_c = 11.6$ sec**

Figure 2: Effect of Floor Collapse Energy on Total Collapse Times

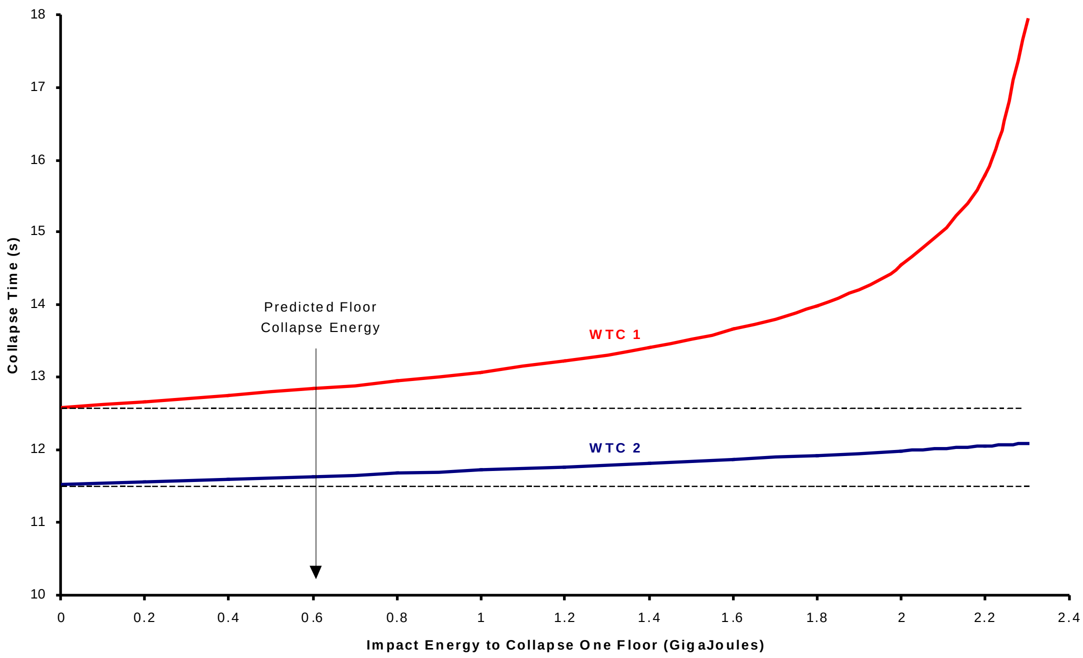
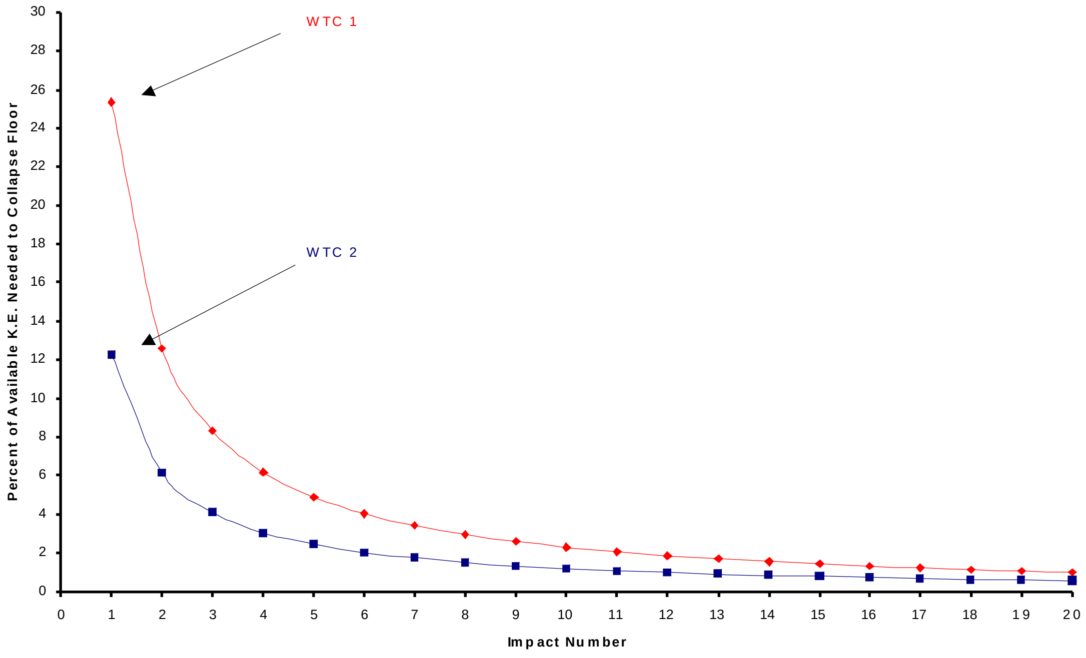


Figure 2 shows that t_c is quite insensitive to the value selected for E_1 up to $E_1 \approx 2.0 \times 10^9$ J. Thus, even if E_1 was *twice* as large as our estimated value of 0.6×10^9 J, t_c would only increase by about 0.5 seconds, (See additional comment on this case below). Considerations such as these lead to the conclusion that a *relatively large increase* in E_1 only produces a *small increase* in the collapse time t_c, providing E_1 is less than 50 % of the kinetic energy delivered to the floor. Our estimate of E_1 places it at 25 % of the initial impact kinetic energy for WTC 1 and 12 % of the impact kinetic energy for WTC 2. Figure 3 shows how rapidly the ratio E_1/T_i, (collapse energy to available kinetic energy), declines from each successive floor collapse. Hence it is not surprising that inclusion of E_1 in our momentum transfer theory increases the calculated collapse times by less than 0.5 seconds.

Because uncertainties also exist in our knowledge of the precise mass of the WTC, M, it is worth noting that the momentum transfer calculations of t_c are also relatively insensitive to the value used for M. To show this, collapse time calculations were run using a fixed value of E_1 of 1×10^9 J and a mass, M, varied in increments of 50,000,000 kg between 510,000,000 kg and 300,000,000 kg. The resulting collapse times varied by less than 0.8 seconds confirming that the collapse time is indeed quite insensitive to M over a wide range. Upon reflection, this is not so surprising since this result is precisely what Galileo showed in his famous experiment at Pisa. Thus, if there is no resistance to the collapse, the Towers would fall in 9.2 seconds - *regardless of their mass!* However, the mass of a tower *does* affect t_c when resistance has to be overcome, but in this case the resulting increase in t_c is obviously mostly dependent on E_1.

It is finally worth noting that because E_1 is a small fraction of the available kinetic energy, the WTC collapse times would not substantially increase even if we allow for the simultaneous crushing of *two* floors – the floor impacted by the falling mass *and* the floor just above the lowest floor of the falling mass. For example, this would involve the following 1st stage collapse sequence for WTC 1: the floors from 110 to 96 (= 14 floors) collapse as a single block onto floor 95, causing the immediate collapse of floor 95 onto floor 94 *and* floor 97 onto 96 and so on. Interestingly, the momentum transfer equations for this mode of collapse are identical to those for the simpler collapse sequence proposed in Section 3.0.

Figure 3: Floor Collapse Energy as a Percentage of Kinetic Energy Available Per Impact



<u>The Question of the Fires</u>

One aspect of the WTC collapse events that we have not, as yet, included in our calculation is the destructive energy inputs from the explosion and combustion of the fuel carried by each impacting aircraft. Let us therefore briefly consider jet fuel as an additional source of energy in the WTC collapse.

We shall assume that each Boeing 767 aircraft was carrying about 30,000 kg of jet fuel when it crashed. In addition we estimate that for the spectacular fireball that was seen after each aircraft impact, 75 % occurred *outside* the struck tower, and 25 % occurred *inside*. Now, because the heat of combustion of jet fuel is 42 MJ/kg, we might conclude that $\frac{1}{4} \times 30{,}000 \times 42 \times 10^6$ J = 315 Gigajoules of energy was supplied to each WTC tower by the jet fuel. However, it is likely that inadequate mixing of air and fuel occurred inside each tower resulting in a fuel-rich vapor cloud at the time of ignition. It follows that only a small fraction of the available combustion heat was actually dissipated by the jet fuel explosion. A detailed study of hydrocarbon vapor cloud explosions has been published by K. Gugan /9/. This author has shown that for the deflagration of large quantities of flammable liquids, the explosive efficiency or yield is typically only 5 %. This implies that the fireball caused by each WTC aircraft impact involved a 16 Gigajoule acute release of blast energy inside each tower. The jet fuel that remained inside each tower after the initial fireballs burned rapidly but started longer lasting fires and is discussed further below.

K. Gugan's data (See /9/), scaled to 7,500 kg of exploding jet fuel, indicate a value of about 2×10^5 Pa for the maximum overpressure at the center of the jet fuel explosions. This blast overpressure would have caused considerable damage to office furniture, wallboards, ceiling tiles and windows on the impacted floors; however, 2×10^5 Pa of blast pressure is insufficient to have seriously affected the structural steel support columns. This conclusion justifies the exclusion of the jet fuel explosions from our energy transfer calculations.

While on the topic of explosions at the WTC on September 11th, 2001, it is worth considering an observation that some researchers consider to be evidence of the use of explosives in the collapse of the twin towers. For example, E. Hufschmid in his book *Painful Questions* (See Ref/1/) discusses the fact that the collapsing towers spewed out horizontal jets of dust and asks: "*How could* (this dust) *be ejected with such a high velocity that the clouds reached 200 to 400 feet?*" Hufschmid concludes that "*packages of explosives installed on nearly every floor*" must have been used! However, careful consideration of the WTC collapse mechanism offers an alternative explanation for the ejected dust that eliminates the need for explosives.

Thus, based on the dimensions of each WTC tower, there were 10,000 m^3 of "open space" per floor. The collapsing floor acted like a giant piston compressing the air occupying the open space between floor and ceiling. The pressure build-up would have shattered windows almost immediately, expelling the enclosed air. However, the process of collapse would have simultaneously crushed the gypsum wallboard and fiberglass insulation present on every floor and some of this debris would have been expelled also. How fast was this dust cloud expelled? The first collapsing floor fell the 3.7-meter ceiling-to-floor distance in 0.87 seconds and subsequent floors fell much faster. It follows that a volume of dusty air near the center of a collapsing floor traversed a horizontal distance of about 16 meters in 0.87 seconds in exiting the building. This volume therefore had an average expulsion velocity of 66 km or 41 miles per hour. As we have shown, the twin towers ultimately attained a collapse velocity in excess of 50 m/s in which case the lower floors were crushed in 0.074 seconds and dust expulsion velocities approached 778 km or 484 miles per hour!

Why Did the Towers Fall?

We have shown in this report that because of the failure of just one floor, a sequential collapse of all remaining floors was inevitable. This, of course, brings us to the $64,000 question:

What caused the initial floor collapse?

Although some researchers apparently find it difficult to accept, I believe the answer to this question is essentially quite simple:

The initial floor collapse occurred due to the aircraft impact damage and the resulting eccentric loading of the core columns.

Before elaborating on this statement, let us first deal with another potential factor in the twin tower's collapse: the weakening of critical floor supports by heat from the jet fuel fires. While this may have been a contributing factor, I do not believe that we need to invoke anything as extreme as the *melting* of structural steel in the WTC to explain why the towers collapsed. The smoky appearance of the fires suggests that the flames inside each tower were fuel-rich and therefore probably below 900° C. In addition, the structural steel was heated indirectly and entire columns probably never attained temperatures much above 750° C. Nevertheless, ~ 20 % loss of strength is to be expected for steel heated to 550° C, a temperature that may have been reached by *some* WTC core columns.

Returning now to the reasons for the initial floor collapse in the WTC towers we need to examine the forces required to support a particular floor in order to understand how it could collapse. For example, consider the support columns at the 80th floor of WTC 2. These columns experienced a *downward* force equal to $M_{30} \times g$, where M_{30} is the mass of thirty floors. Hence the downward force at the 80th floor is $30 \times 4.64 \times 10^6 \times 9.81$ Newtons = 1366 MN. As noted in Section 4.2, the 236 perimeter and 47 core support columns have an effective cross-sectional area of $(236 \times 0.0184 + 47 \times 0.1236)$ m^2 = 10.15 m^2. If we assume that the support columns are fabricated from high strength steel with an effective compressive yield stress, σ_y, of 400 MN/m^2 we conclude that the supports at a given floor would fail if the downward compressive load exceeded about

4000 MN. Thus for the above example we see that the 80th floor has a collapse safety factor of about 3 which is well within acceptable limits for modern high-rise buildings.

Now consider the 80th floor of WTC 2 *after* the aircraft impact. About 20 % of the support columns have been destroyed and another 10 % may have been buckled to some degree. The safety factor for collapse of the 80th floor is now only a little over two, but apparently still sufficient to sustain the building almost indefinitely. However, the damage to the twin towers was *asymmetric* so that the post-impact gravity load above the impacted floor was no longer uniformly distributed. For WTC 2 the load normally carried by the first two rows of columns in the southern-most corner of the core would have shifted to undamaged columns in adjacent rows after the aircraft impact. The upper block of 30 floors would have leaned (imperceptibly) to the south and would have created a bending moment along the mid-core rows (comprising about ten columns). Immediately prior to collapse the effective cross-sectional area supporting the 30 floors above the 80th floor would have been only about $\{(½ \times 200 \times 0.0184) + 10 \times 0.1236)\}$ $m^2 = 3.1$ m^2. We would therefore expect the floor support to fail because a reaction force of 400×3.1 MN = 1240 MN is significantly below the 1366 MN required to sustain the structure.

It should be noted that the critical loads estimated above have been treated as purely *compressive loads*. However, it is well known from the theory of columns (See, for example Chapter IX of Ref /10/), that even *marginally* eccentric loads are capable of producing very large lateral deflections. If the deflection becomes large, the bending moment and the stresses are also large and the elastic limit is exceeded resulting in column failure. It is suggested that in the minutes after the aircraft impacts, localized stresses were created within the tower's support structures that gradually established the conditions for the failure of at least one floor. The development of critical stresses in the damaged areas of the towers will probably never be known in any great detail. However, the conservative estimates of the magnitude of asymmetric loadings given above suggest that *aircraft damage alone was sufficient to initiate a total collapse of the buildings.*

One final comment with regard to the causes of the WTC collapse is worth making. It has been stated many times that the WTC towers were designed to withstand the impact of a commercial aircraft with the weight and speed specifications of a Boeing 707 or 767. Indeed, a few observers find solace in the fact that both towers survived the impacts for as long as they did. This has led some commentators to assume that simply because the WTC towers were <u>*claimed*</u> to be plane crash resistant their collapse <u>*must have*</u> been triggered by explosives or some other destructive devise. The logical flaw in this argument stems from the difference between what one may *claim* to be true and what subsequently *proves* to be true. Evidently the design calculations on which the crashworthiness of the WTC was based were in error. The infamous *Titanic* disaster was also an accident that "could not happen". However, even though the *Titanic* failed to live up to its "unsinkable" billing, no one has seriously proposed that its loss was due to explosives hidden somewhere on board that ill-fated ship. Unfortunately the twin towers had an Achilles' heel that was finally revealed to the world on September 11th, 2001.

7.0 CONCLUSIONS

• An analysis of the energetics of the WTC collapse events has shown that the kinetic energy of the aircraft collisions and the subsequent gravitational energy released by the descending blocks of floors were quite sufficient to destroy the twin towers in the manner observed. The use of explosive devices in either of the two towers is not necessary to explain the collapse events and is considered to be *highly unlikely*.

• The times calculated for the collapse of WTC 1 and WTC 2 show good agreement with the observed collapse times verifying the basic assumptions of the momentum transfer model used in the calculations.

• The calculated times represent the *minimum* theoretical times of building collapse. If shorter times are to be physically achieved they must involve an unknown additional source of energy acting in a downward direction. Such a source of energy does not appear to have been involved in the collapse of the twin towers.

• The kinetic energy of the collapse events was sufficient to crush the WTC floor concrete in both towers to particles 100 μm in diameter, or smaller, which is consistent with the observed WTC debris particle size distribution.

• From a consideration of the strength of the WTC columns, and the effective area of support they provided, it is demonstrated that the conditions necessary for the initial floor collapse were initiated by the aircraft impacts and made irrevocable by the subsequent eccentric loading of the core columns. The fires that were initiated by the jet fuel spilled within the towers certainly weakened steel in localized areas in the impact zones. However, it is suggested that the total collapse of both towers would have occurred *even without the jet fuel fires*.

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APPENDIX 1 : WORLD TRADE CENTER FACTS

HEIGHT: Tower 1 (North) = 1368 ft = 417 m
Tower 2 (South) = 1362 ft = 415 m

NUMBER OF FLOORS = 110

Allowing for floor thickness → 12.1 ft (3.7 m) height from floor to ceiling

WIDTH OF EACH TOWER = 209 ft (63.7 m)

GROSS FLOOR AREA = 43,681 sq ft = 4058 m^2

DIMENSIONS OF CENTER CORE = 89 ft (27.1 m) × 139 ft (42.4 m)

AREA OF CENTER CORE = 12,371 sq ft = 1,149 m^2

NET FLOOR AREA = 31,310 sq ft = 2909 m^2

VOLUME OF OFFICE "AIR-SPACE" PER FLOOR = 10996 m^3

USABLE OFFICE SPACE PER FLOOR = 20,550 sq ft = 1909 m^2

WEIGHT OF EACH TOWER = 510,000,000 kg

WEIGHT OF EACH FLOOR = 4,636,363 kg

TIME TO FALL, $t = \sqrt{(2s/g)} = \sqrt{(832/9.81)} = 9.2$ seconds

MAX IMPACT VELOCITY = $V_M = gt = 9.81 \times 9.2 = 90.25$ m/s = 202 mph

Each floor had a layer of lightweight concrete 4 inches (0.1 m) thick.

The net floor area was 2909 m^2, giving 290.9 m^3 of concrete per floor.

If the density of the WTC concrete was 1500 kg/m^3 we have 110 × 290.9 × 1500 kg

= 48,000,000 kg of concrete flooring per tower

(N.B. Some concrete was also used in the central core of each tower.)

Each tower also had 96,000,000 kg of structural steel

APPENDIX 2: WTC CONCRETE CRUSHING CALCULATIONS

The potential energy stored in one WTC tower = 1.0×10^{12} J

(This result is assuming mass of one WTC tower = 510,000,000 kg)

The WTC debris pile contained concrete crushed to particles in the size range 20 μm to 100 μm. Let's consider 60 μm concrete particles as an example.

What is the energy needed to crush the WTC concrete to a 60 μm powder?

Surface area of one 60 μm particle, (considered to be cubic) = $6 \times 60 \times 60 \times 10^{-12}$ m^2

Mass of one 60 μm particle = 1500 kg/m^3 $\times 60 \times 60 \times 60 \times 10^{-18}$ m^3 = 3.24×10^{-10} kg

Surface area of 1 kg of crushed concrete = ($6 \times 60 \times 60 \times 10^{-12}$ m^2) / (3.24×10^{-10} kg)

$$= 67 \text{ m}^2$$

G_f = Fracture Energy = Energy needed to create a unit area of fracture surface

Typical fracture energy of concrete is ~ 100 Joules/ m^2

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Hence fracture energy required to produce 1 kg of 60 μm powdered concrete (having an effective surface area of 67 m^2) = 100 Joules/ m^2 $\times$ 67 m^2 = 6700 J

One tower contains 48,000,000 kg of concrete, hence energy to crush all of this concrete to 60 μm particles:

$$= 48{,}000{,}000 \times 6700 \text{ J} = 3.2 \times 10^{11} \text{ J}$$

Addendum:

The following material has been prepared since the completion of our *Energy Transfer in the WTC Collapse Events of September 11th, 2001*, in March 2005.

An Assessment of the Time Delays Involved in the WTC Collapse Events

The “true” collapse time of a WTC Tower is made up of two components, the free-fall time (in a vacuum), t_f, plus time delay corrections associated with overcoming the retarding forces acting on the collapsing floors, t_d. Thus:

$$t_c = t_f + t_d$$

The retarding forces are of three types: air resistance, and what we shall call the crushing and bending resistance of a WTC Tower. Crushing resistance refers to the forces involved in fracturing and crushing the concrete floors. Bending resistance refers to the forces involved in bending structural steel components, which could be whole columns or individual bolts and welds, to the point of failure by fracture. We will now consider these three retarding forces in detail:

(i) Air Resistance

The downward force, F_D, acting on an object in free-fall is given by:

$$F_D = Mg$$

Where M is the mass of the object in kg, and g is the acceleration due to gravity equal to 9.81 m/s^2.

The upward force acting on an object in free-fall is the air resistance, F_U, given by:

$$F_U = ½ \rho A C_d v^2$$

Where ρ is the density of air equal to 1.225 kg/m^3 at sea level; A is the effective area of the object in m^2; C_d is a dimensionless quantity known as the coefficient of drag that has values in the range 0.5 to 1.0; v is the instantaneous descent velocity in m/s. (The so-called *terminal velocity* is attained when $F_D = F_U$)

Let’s apply these equations to the WTC 1 case of the upper block of 14 floors falling onto the floor below:

$$M = 14 \times \{510,000,000/110\} = 6.49 \times 10^7 \text{ kg}$$

Hence,

$$F_D = 6.37 \times 10^8 \text{ Newtons}$$

We will take A, the effective area of the falling block of floors, to be the net geometrical area of a WTC floor or 2909 m^2. We will assume a drag coefficient of 0.67. We consider two cases: The air resistance at the lowest impact velocity of 8.52 m/s and the air resistance at the highest impact velocity of 90.2 m/s. For the first case we find,

$$F_U = ½ \times 1.225 \times 2909 \times 0.67 \times 72.59 \text{ kg m/s}^2 = 8.67 \times 10^4 \text{ Newtons}$$

Clearly, for this case, air resistance is negligible compared to the downward force of gravity.

For the second case we have,

$$F_U = ½ \times 1.225 \times 2909 \times 0.67 \times 8136 \text{ kg m/s}^2 = 9.71 \times 10^6 \text{ Newtons}$$

Thus the air resistance force is about 100 times greater than the first case because of the higher velocity; but even this value of F_U is only 1.5 % of the downward accelerating force. We conclude that air resistance is not a significant factor in the collapse of the WTC.

(ii) <u>Crushing Resistance</u>

The crushing resistance correction to the descent time refers to the resistance offered by the 4-inch (10 cm) layer of concrete on each WTC Tower floor. Consider the example of the WTC 1 collapse involving the descent of 14 blocks of floors a distance of 3.7 meters onto the floor below. At the moment of impact the lower floor is already subject to a compressive force, known as the static load, F_{st}, equal to $M_{14} \times g$, where M_{14} is the mass of 14 floors and g is the acceleration due to gravity. Hence, $F_{st} = 6.37 \times 10^8$ Newtons.

After impact, the loading of the lower floor concrete increases for a finite time interval we call Δt, at which point the yield strength of the concrete is reached and the concrete fails by fracture. For simplicity we shall assume that the compressive force acting on the concrete increases linearly up to a value F_y given by,

$$F_y = \sigma_y A$$

where σ_y is the yield strength of concrete in N/m^2, and A is its surface area in m^2. Thus the compressive force acting on the concrete has an *average* intensity of ½ F_y Newtons for a time Δt, and imparts a change of momentum to the falling mass of 14 floors given by:

$$M_{14} \Delta v = ½ F_y \Delta t$$

In which case:

$$\Delta v = (\sigma_y A \Delta t) / 2 M_{14}$$

The quantity Δv is a measure of the loss in kinetic energy of the falling block of floors, which in turn is related to the energy, E_c, expended in fracturing and crushing the concrete on one floor. This may be expressed mathematically as:

$$E_c = \tfrac{1}{2} M_{14} v^2 - \tfrac{1}{2} M_{14} (v - \Delta v)^2$$

where v is the impact velocity.

If Δv is small compared to v it follows that to a good approximation:

$$E_c = M_{14} v \Delta v$$

hence, substituting for Δv, and solving the equation we have

$$E_c = \tfrac{1}{2} v (\sigma_y A \Delta t)$$

In order to proceed further with this analysis we note that experimental values of Δt for the fracture of concrete are available in the published literature, /1, 2, 3/. The published values fall in the range 0.5 – 5 milliseconds, hence we will take 5 ms as an upper limit value for Δt. Also, for the WTC concrete we will take σ_y to be 5 MN/m^2 and A to be 2000 m^2. Hence,

$$E_c = \tfrac{1}{2} v \times 5 \times 10^6 \times 2000 \times 5 \times 10^{-3} \text{ Joules}$$

or,

$$E_c = v (2.5 \times 10^7) \text{ Joules}$$

As an example of the use of this equation we note that the velocity of the first impact in WTC 1 is 8.52 m/s, in which case $E_c = 2.13 \times 10^8$ Joules or about 1/3rd of the estimate of the energy required to collapse one floor made in our *Energy Transfer in the WTC Collapse* report.

In our WTC report we also show that the fracture energy of concrete is typically ~ 100 J/m^2 and the effective surface area of 1 kg of concrete particles with average diameter d microns, d(μm), is equal to {4000/ d(μm)} m^2. Since the mass of concrete per WTC floor is 4.36×10^5 kg, it follows that the energy expended in crushing the concrete on one floor to particles of diameter d(μm) is $4.36 \times 10^7 \times$ {4000/ d(μm)} Joules. But this is our previously defined quantity E_c; hence we arrive at an equation relating the size of the crushed concrete particles to the impact velocity:

$$4.36 \times 10^7 \times \{4000/ d(\mu m)\} = v(2.5 \times 10^7)$$

or,

$$\mathbf{d(\mu m) = 6976/v}$$

This result differs from our *Energy Transfer in the WTC Collapse* report, which simply assumed that the concrete was pulverized to a constant particle size for each and every floor. Our new formalism shows that the particle diameter is in fact proportional to 1/v, or equivalently, the specific surface *area* of the particles is proportional to v^2. Now, since the impact energy is $\frac{1}{2} M_{14} v^2$, we have the physically appealing result that the specific surface area of the particles increases as the impact energy increases, a behavior reported for rock fragmentation under high speed impact loading /4, 5/.

It is of interest to apply our formulae to the WTC 1 collapse and calculate the diameter of concrete particles produced by the impact of the upper block of 14 floors on a number of representative lower floors:

Floor impact	**Concrete Particle Diameter (μm)**
1^{st}	819
10^{th}	259
50^{th}	116
100^{th}	82

Thus we see that most of the WTC 1 concrete is crushed to particles less than 200 μm in diameter. The somewhat smaller particle size observed in WTC debris samples may be accounted for by contributions from the more easily crushed gypsum wallboard and fiberglass insulation. Chemical and particle size analyses reported by researchers such as P.J. Lioy and G. Meeker provide data on the composition of WTC dust and confirm that concrete accounts for only about 40 % of its total mass.

(iii) Bending Resistance

In our *Energy Transfer in the WTC Collapse* report we show that the energy expended in collapsing one floor, which we called E_1, is approximately equal to 0.6 gigajoules. We also show that the total collapse time, t_c, of each WTC Tower is relatively insensitive to the value of E_1 for values up to about 2 gigajoules. We will now consider in detail the connection between E_1, t_c and the bending/fracture resistance of the structural steel in a WTC Tower.

For the block of 14 floors initiating the WTC 1 collapse:

$$E_1 = \frac{1}{2} M_{14} v^2 - \frac{1}{2} M_{14} (v - \Delta v)^2$$

where v is the impact velocity in m/s and Δv is the velocity decrement due to impact, a measure of the kinetic energy expended in bending and fracturing the structural steel columns supporting the impacted floor.

To fully understand the relationship between E_1 and Δv, we note that the kinetic energy loss involved in the first WTC 1 floor collapse is work, W, performed on the descending block of floors by a steadily increasing resistance force. Hence we write:

$$W = \tfrac{1}{2} F_y \times d$$

where d is a characteristic distance and F_y is the effective yield strength of the steel support structures on one WTC floor. The factor ½ is included because ½ F_y is the *average* force acting on the impacted floor.

F_y , the maximum value of the retarding force, induces a change in velocity Δv over a time interval Δt. It follows that:

$$E_1 = \text{Work, } W = \tfrac{1}{2} F_y \times d = \tfrac{1}{2} F_y \times \Delta t \, \Delta v$$

Thus the collapse energy E_1 depends on the product of three factors:

(i) F_y, the effective loading force at the instant of structural failure.

(ii) Δt, the time interval during which F_y is active.

(iii) Δv, the velocity decrement of the descending block of floors.

It is possible to calculate F_y directly using a formula from the theory of elastic strain energy, (See for example Ref /6/):

$$F_y = F_{st}\{1 + \sqrt{[\,1 + (2hAE/LM_{14}g)\,]}\}$$

Where:

h is the drop height of the upper block of floors = 3.7 m
A is the effective area of the core support columns = 5 m^2
E is Young's Modulus for structural steel = 180×10^9 N/m^2
L is the effective length of the core support columns = 10 m

From which we calculate: **$F_y = 2 \times 10^{10}$ Newtons.**

F_y is therefore about 33 times greater than the downward static force F_{st} of 6.37×10^8 Newtons normally acting on the 95^{th} floor. The fact that F_y is so much larger than F_{st} is to be expected in situations of intense dynamic loading such as the collapse of the WTC. It must be remembered, however, that although F_y represents a very large force, it acts over a very short time interval, Δt, as we show below. For now we note that published data on the response of steel columns to axial impact indicate that Δt is in the range 2 – 20 ms /7/.

With the magnitude of F_y determined in this way we are in a position to quantify the retarding effects of the WTC floor supports by including suitable correction terms in our momentum transfer calculations. First we note that, upon impact with the floor below, the *net* force, F_n, acting on a descending block of floors of mass M is no longer Mg, but

$$F_n = Mg - \tfrac{1}{2} F_y$$

Thus the acceleration, *a*, experienced by M is less than g by an amount $F_y/2M$. We shall assume that this reduced force acts over a distance d. Additionally, if u is the initial (impact) velocity and v is the reduced velocity attained after a distance d:

$$v^2 - u^2 = 2ad$$

and,

$$a = g - F_y/2M$$

Hence,

$$d = (v^2 - u^2)/(2g - F_y/M)$$

and the time increment, Δt, is given by:

$$\Delta t = 2d/(u + v)$$

We have used the above equations to amend our first-order momentum transfer calculations to incorporate the time delay involved in the bending/fracturing of the steel support structures on each WTC floor. An example of the computational results for the first five floor failures of WTC 1 is given in Table 1.

Table 1: Calculated Initial Phase of Collapse Profile for WTC 1 Including Time Delay from Bending/Fracturing Steel Support Structures (E_1 = 1 GJ)

Height of Floor 110 h(m)	Initial Velocity u (m/s)	Final Velocity v(m/s)	Force Action Distance d(m)	Time Delay Δt (s)	Elapsed Time t(s)
416	0	0	-	-	0
412.3	7.99	6.07	0.1078	0.0153	1.2192
408.6	10.99	9.77	0.1084	0.0104	1.7017
404.9	13.13	12.19	0.1089	0.0086	2.0492
401.2	14.83	14.05	0.1100	0.0076	2.3399
397.5	16.25	15.57	0.1106	0.0069	2.5973

Table 1 shows that the bending/fracturing of the steel supports delays the descent of WTC 1 by only 15.3 milliseconds for the first impact and by a steadily decreasing time interval thereafter. The full calculation for the total collapse of the Twin Towers shows that inclusion of the delay from the bending/fracturing of the steel supports adds only about 0.5 seconds to t_c for WTC 1 and 0.3 seconds to t_c for WTC 2.

Also of interest in Table 1 is the small, relatively constant, value ~ 0.11 meters calculated for the distance d over which the retarding effect of the steel supports is active. This distance is considerably smaller than the 3.7-meter drop height. However, this result is not surprising when the condition of the failed steel support columns is considered. Examination of photographs of the remains of the Twin Towers after the events of 911 shows that most of the core and perimeter columns in the debris field were *not* severely buckled. On the other hand, fracture of A325 bolts at the column splice plates appears to have been a common failure mode. This suggests that the strength of the column connecting bolts was *not* well matched to the strength of the column members they were designed to hold. It would therefore appear that the A325 connecting bolts failed before the full strength of many of the columns was realized, explaining why most of the columns in the rubble pile were not severely buckled.

Post Script:

An Observational Confirmation of the Calculated WTC Collapse Times

In concluding our re-evaluation of the WTC collapse times we have also evaluated t_c *directly* from measurements based on a number of available videos of the collapse of WTC Buildings 1 and 2. The videos selected for these measurements involved unobstructed views of the top of the Tower in question and were in real time. However, the descent of the Tower was followed using a "freeze-frame" technique whereby the video was advanced and paused at one-second intervals. A clear plastic ruler was mounted against the TV monitor screen and the *screen* distance advanced by a selected feature of the Tower was measured and recorded. Appropriate features used for these measurements included: the topmost floor of the Tower, the radio mast (for WTC 1), a burning floor, etc. The *screen* distance in cm was converted to a *real* distance in meters using known dimensions of features of the Twin Towers such as the height of one floor (= 3.7 m) or the width of each Tower (= 63.7 m).

This measurement technique yielded very consistent values for the downward movement of the selected features on the Twin Tower over the first four or five seconds of collapse. Thereafter, the selected feature invariably became obscured in the expanding cloud of dust and debris that was associated with the collapse of each Tower. Nonetheless the data obtained over the first four seconds are quite sufficient to show that WTC 2 fell significantly faster than WTC 1. What is more, the measured downward movement of each Tower may be compared with our *calculated* values for the same downward movements. The results of such a comparison are shown in Figure 1. This figure uses calculations with E_1 set to 0.8 GJ. This value of E_1 was selected to include 0.2 GJ of collapse energy to crush the concrete floors. The total collapse times were then calculated as follows:

WTC 1: t_c = 13.48 s ; WTC 2: t_c = 12.07 s

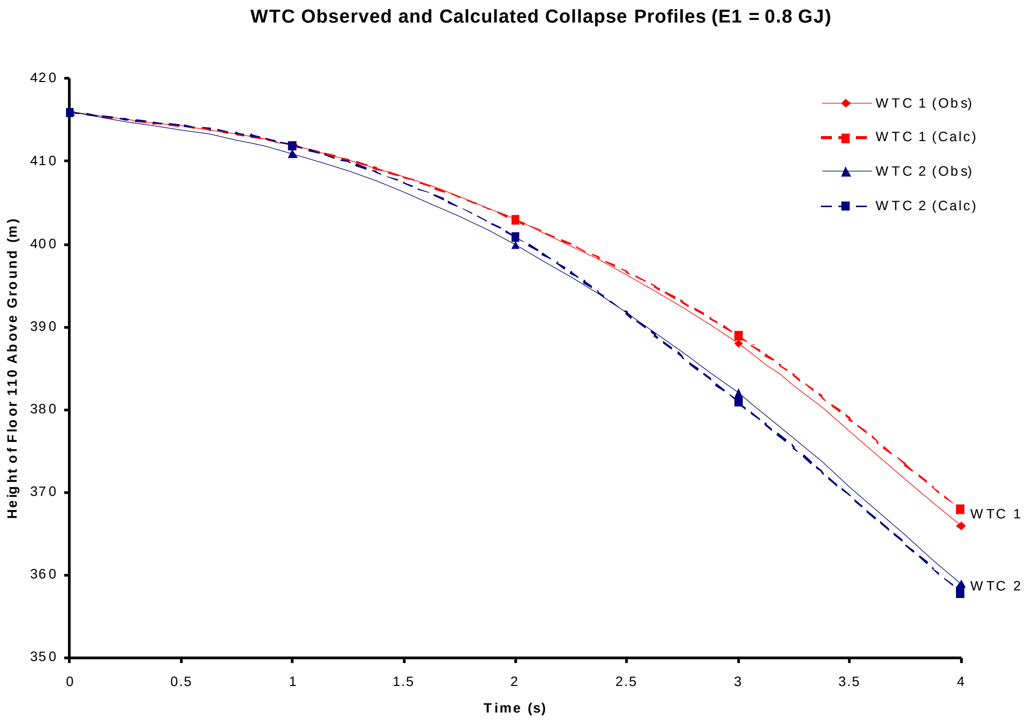

Figure 1 shows very good agreement between the observed behavior of WTC1 and WTC 2 and the calculated result with $E_1 = 0.8$ GJ.

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